BLUNT BODIES IN A STREAM OF NONEQUILIBRIUM-IONIZED RADIATING GAS

N. V. Leont'eva, Yu. P. Lun'kin, and A. A. Fursenko

UDC 533.6.011

A supersonic stream of an inviscid monoatomic nonequilibrium-ionized radiating gas around blunt bodies is analyzed in this study. The results are illustrated by the example of an argon stream around spherically blunted bodies.

1. Kinetic Model

When analyzing a supersonic stream of argon around a blunt body, within the region between the body surface and the front of a shock wave one considers collision ionization through an excited level and energy transfer from heavy particles to the electron gas, as a result of elastic collisions between electrons and ions or atoms; one also considers photoionization from the fundamental level, i.e., one considers altogether the following reactions:

$$A + M \rightleftarrows A^* + M, \qquad A^* + M \rightleftarrows A^+ + M + e,$$
$$A + A \rightleftarrows A + A^+ + e, \qquad A + h\nu \rightleftarrows A^+ + e,$$

where A and A* denote atoms in the fundamental state and in an excited state, respectively; A^+ denotes a single-charge ion; e denotes an electron; h denotes a photon; and M denotes either A or e.

It is assumed that the excited states of atoms enter into equilibrium with free electrons [1]. The rate at which an excited state becomes occupied may be assumed equal to zero, moreover, and the rate of collision ionization (which is determined by the difference between excitation rate and deexcitation rate) may be expressed as follows:

$$\dot{n}_+ = \dot{n}_{ea} + \dot{n}_{aa} = [K_e(1,\ 2)\, n_e n(1) - K_e(2,\ 1)\, n(2)\, n_e] + [K_a(1,\ 2)\, n_a n(1) - K_a(2,\ 1)\, n(2)\, n_a], \tag{1}$$

where $n_a = n(1) + n(2)$.

We will now derive an equation for the rate of ionization by an electron impact. Since during equilibrium all forward processes are balanced by the respective reverse processes, hence

$$K_e(1,\ 2)\, n_e n(1) = K_e(2,\ 1)\, n(2)\, n_e \tag{2}$$

and thus

$$K_e(2,\ 1) = K_e(1,\ 2)\,\frac{g_1}{g_2}\,\exp\left(\frac{T_{ex}}{T_e}\right). \tag{3}$$

From (3) we have

$$\dot{n}_{ea} \doteq K_e(1,\ 2)\, n_e n(1)\left[1 - \frac{n(2)}{n(1)}\,\frac{g_1}{g_2}\,\exp\left(\frac{T_{ex}}{T_e}\right)\right]. \tag{4}$$

Considering that excited states enter into equilibrium with free electrons, we have [2]

A. F. Ioffe Institute of Physics and Engineering, Academy of Sciences of the USSR, Leningrad.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 25, No. 4, pp. 681-687, October, 1973. Original article submitted February 1, 1973.

$$\frac{n_e^2}{n(2)} = \frac{g_i}{g_2} \frac{2(2\pi m_e kT_e)^{3/2}}{h^3} \exp\left(-\frac{T_{i2}}{T_e}\right), \tag{5}$$

where $T_{i2} = T_j - T_{ex}$ denotes the ionization potential of an excited state. Since $T_{ex} \approx T_j$, hence

$$n(1) \approx n_a = (1-\alpha)\frac{\rho}{m_a}. \tag{6}$$

Introducing $\alpha_E(T_e)$ as Saha's solution and performing the necessary transformations, we finally obtain

$$\dot{n}_{ea} = \alpha(1-\alpha)\left(\frac{\rho}{m_a}\right)^2 K_e(1,2)\left[1 - \frac{\alpha^2}{1-\alpha}\frac{1-\alpha_E(T_e)}{\alpha_E^2(T_e)}\right]. \tag{7}$$

The rate coefficient $K_e(1,2)$ introduced in [3] is

$$K_e(1,2) = 2C_e\sqrt{\frac{2}{\pi m_e}}(kT_e)^{3/2}\left(\frac{T_{ex}}{T_e}+2\right)\exp\left(-\frac{T_{ex}}{T_e}\right), \tag{8}$$

where $C_e = 4.4 \cdot 10^{-3}$ m^2/J [1].

For an atom–atom ionization passing through an excited level, the expression for $\dot{n}_{aa}$ is derived analogously to the expression for $\dot{n}_{ea}$:

$$\dot{n}_{aa} = (1-\alpha)^2\left(\frac{\rho}{m_a}\right)^2 K_a(1,2)\left[1 - \frac{\alpha^2}{1-\alpha}\frac{1-\alpha_E(T_e)}{\alpha_E^2(T_e)}\exp\left(\frac{T_{ex}}{T_a}-\frac{T_{ex}}{T_e}\right)\right]. \tag{9}$$

The rate coefficient $K_a(1,2)$ in [3] is

$$K_a(1,2) = 2C_{aex}\sqrt{\frac{1}{\pi m_a}}(kT_a)^{3/2}\left(\frac{T_{ex}}{T_a}+2\right)\exp\left(-\frac{T_{ex}}{T_a}\right), \tag{10}$$

where $C_{aex} = 1.56 \cdot 10^{-5}$ m^2/J [4].

For an atom–atom ionization directly from the fundamental state, the expression for n_{aa} is

$$\dot{n}_{aa} = (1-\alpha)^2\left(\frac{\rho}{m_a}\right)^2\frac{2C_{ag}}{(\pi m_a)^{1/2}}(kT_a)^{3/2}\left(\frac{T_j}{T_a}+2\right)\exp\left(-\frac{T_j}{T_a}\right), \tag{11}$$

where $C_{ag} = 7.5 \cdot 10^{-6}$ m^2/J [5].

We note that the expressions derived in [3] for n_{ea} and n_{aa} as well as the source function in the equation of radiative energy transfer contain an error. Such erroneous formulas were also used later in [6, 7]. Our analysis has shown this error to be appreciable. For instance, the radiation flux profile calculated with the source function according to [3] has two peak points [6]. Calculations according to the correct formula yields, as was to be expected, only one peak.

An electron gas receives energy from heavy particles during electron–ion and electron–atom collisions. These processes occur at rates

$$\omega_{ie} = \left(\frac{\rho}{m_a}\right)^2\alpha^2\frac{e^4}{m_a}\left(\frac{8\pi m_e}{kT_e}\right)^{1/2}\left(\frac{T_a}{T_e}-1\right)\ln\left[\frac{9(kT_e)^3}{4\pi n_e e^6}+1\right], \tag{12}$$

$$\omega_{ae} = \left(\frac{\rho}{m_a}\right)^2\alpha(1-\alpha)\frac{24\sigma}{m_a}\left(\frac{2m_e}{\pi}\right)^{1/2}\left(\frac{T_a}{T_e}-1\right)(kT_e)^{5/2}\left[1-\frac{C_1}{T_e}+\frac{C_2}{T_e(1+C_3T_e)^3}\right], \tag{13}$$

where $\sigma = 0.14$ m^2/J, $C_1 = 0.18 \cdot 10^4$°K, $C_2 = 0.28 \cdot 10^4$°K, and $C_3 = 1.8 \cdot 10^{-4}$°K^{-1}.

2. Fundamental System of Equations

In the notation of [6] these equations are

$$\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{\rho}\frac{\partial \rho}{\partial r} + \frac{v}{\rho r}\frac{\partial \rho}{\partial \theta} + \frac{2u}{r} + \frac{v}{r}\operatorname{ctg}\theta = 0, \tag{14}$$

$$u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{15}$$

$$u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + \frac{uv}{r} = -\frac{1}{\rho r} \frac{\partial p}{\partial \theta}, \tag{16}$$

$$\left(\rho u \frac{\partial}{\partial r} + \frac{\rho v}{r} \frac{\partial}{\partial \theta}\right)\left[\frac{V^2}{2} + \frac{3}{2} R(T_a + \alpha T_e) + \alpha R T_j\right] = -\frac{\partial q}{\partial r}, \tag{17}$$

$$\left(\rho u \frac{\partial}{\partial r} + \frac{\rho v}{r} \frac{\partial}{\partial \theta}\right)\left[\frac{5}{2} R T_e \alpha\right] = \omega_{ae} + \omega_{ie} - k T_j \dot{n}_{ea}$$

$$+ \frac{3}{2} k T_0 \dot{n}_{aa} + k T_e \dot{n}_{rad} - e n_e \vec{E} \cdot \vec{V}, \tag{18}$$

$$\rho u \frac{\partial \alpha}{\partial r} + \frac{\rho v}{r} \frac{\partial \alpha}{\partial \theta} = m_a (\dot{n}_{aa} + \dot{n}_{ea} + \dot{n}_{rad}), \tag{19}$$

$$\frac{m_e}{m_a} \rho_i (\vec{V} \nabla) \vec{V} = -\nabla p_e - e n_e \vec{E}, \tag{20}$$

$$p = \rho R (T_a + \alpha T_e). \tag{21}$$

Here T_0 denotes the temperature of electrons released as a result of atom–atom collisions. Having no sufficiently reliable data available on the energy of electrons released by such reactions, one has assumed that $T_0 = T_{ew}$.

Unlike in [6], the system (14)-(21) here accounts for the polarization field which impedes the dissociation of charges [8]. These equations must be supplemented by the equation of radiative energy transfer, given in [9, 10] in the approximation of a locally uniform layer.

The boundary conditions for Eqs. (14)-(21) are analogous to those in [6, 10], except for the electron temperature which takes into account an adiabatic compression of the electron gas through the jump:

$$T_{ew} = \left(\frac{\rho_w}{\rho_\infty}\right)^{\gamma-1} T_{e\infty}. \tag{22}$$

It has been assumed that the ionization coefficient α remains continuous during passage of a shock wave. The magnitude of α before a shock front will be discussed later on.

The fundamental system of equations was rewritten in θ, ξ coordinates ($\xi = (r - r_b)/(r_w - r_b)$) and transformed into a dimensionless one, as in [10]. It was solved on a model BÉSM-4 computer by the method given in [10]. Unlike in [6], however, in this study the flow field was calculated not only along the zero streamline but also over the entire subsonic region.

3. Dependence of the Solution on the Choice of Kinetics

In order to explore the effect of various collisions mechanisms of the gasodynamic fields, we considered Eqs. (14)-(21) without taking into account radiation. Calculations have shown that, during atom–atom ionization through an excited state, a change in the initial ionization coefficient α_w from 10^{-5} to 10^{-3} does not affect the gasodynamic fields. With elastic electron–atom collisions taken into account, the relaxation zone becomes slightly shorter.

It has also been found that the gasodynamic parameters of the shock layer change negligibly little during an adiabatic compression of the electron gas (formula (22)), when α and T_e before the wave front are low. When the electron temperature T_e before the wave front is high [11], however, then accounting for an adiabatic compression may correct the results substantially.

4. Effect of Radiation

The rate of photoionization from the fundamental level $\dot{n}_{rad}$ and the expression for radiative flux q can be found in [9, 10]. Our calculations were based on approximating the absorptivity $\varkappa$ as a "ladder" function of the frequency ν. It has been found that the radiative flux and the gasodynamic fields depend only weakly on the number of "steps" in the approximation of $\varkappa(\nu)$.

According to calculations, the radiation flux profile q depends strongly on the Mach number and on the pressure. Thus, for a body with a radius of 0.04 m under a pressure of $p_\infty = 150$ N/m^2, an increase

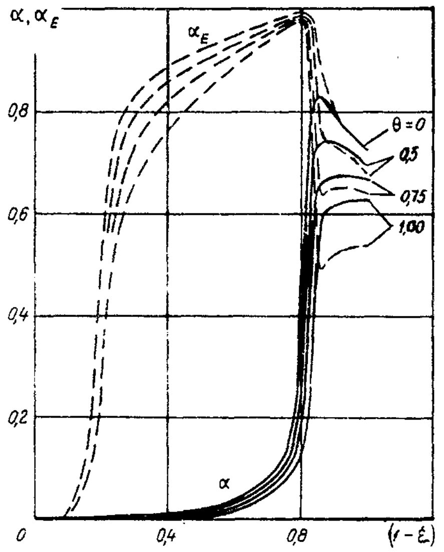

Fig. 1

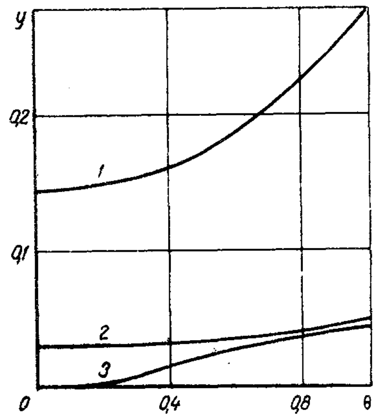

Fig. 2

Fig. 1. Profile of the nonequilibrium ionization coefficient α and of the equilibrium-ionization coefficient α_E along rays θ in a shock layer: α, α_E, ξ) dimensionless quantities.

Fig. 2. Shape of shock wave (curve 1), of ionization front (curve 2), and of local-equilibrium surface (curve 3); ordinate y (dimensionless); angle θ, rad.

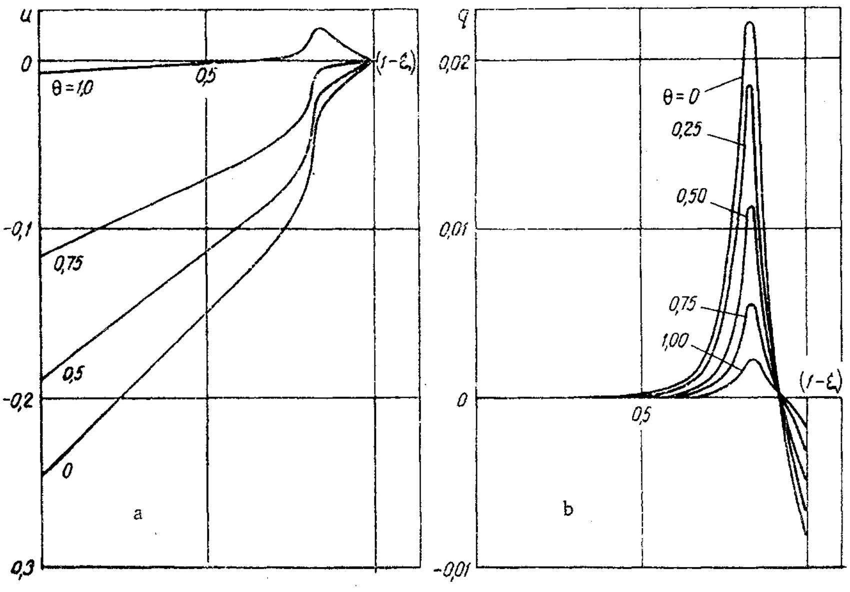

Fig. 3. Profile of the normal velocity component u (a) and of the radiation flux q (b) along rays θ through the shock layer; u, q, ξ) dimensionless quantities; angle θ, rad.

in the Mach number from 30 to 32 causes the maximum thermal flux q to increase from 25,000 to 55,000 kW/m^2; at $M_\infty = 32$ and $p_\infty = 100$, 150, and 200 N/m^2 we have $q_{max} = 27{,}000$, 53,000, and 84,000 kW/m^2, respectively.

5. Variation of the Flow Parameters along the Shock Layer

For calculating the gasodynamic parameters in a stream around a spherically blunted body, we used the approximation of the sought gasodynamic function toward $\xi = \text{const}$. The fundamental system of differential equations was solved for the derivatives with respect to ξ, then integrated along three rays at angles $\theta = 0$, θ_1, and θ_2, respectively. By varying the position of rays θ_1 and θ_2, under the same conditions in the stream, one can obtain the flow pattern in the shock layer precisely enough.

As an example, we considered the parameter profiles along theoretical rays $\theta = 0$, 0.25, 0.50, 0.75, and 1.00 rad in a stream with $M_\infty = 30$, $p_\infty = 150\ N/m^2$, $T_{a\infty} = T_{e\infty} = 300°K$, and $\alpha_W = 10^{-5}$ around a body spherically blunted to a radius of 0.04 m and having an emissivity $\delta = 0.5$. The value of $\alpha_W = 10^{-5}$ was based on the assumption that the entire radiation from the wave front would be absorbed by the oncoming stream with a subsequent release of electrons. The values of $\alpha(\xi, \theta)$ and $\alpha_E(\xi, \theta)$ are shown in Fig. 1. It is quite evident that the ionization coefficient α decreases with larger angles θ. With radiation near the body surface taken into account, then, there appears a radiative entropy layer characterized by a drop in α as well as in the temperatures T_a and T_e. As the gas flows along the shock layer, the equilibrium ionization coefficient α_E according to Saha's equation at temperature T_e decreases. Recombination plays an increasingly important role at larger angles θ.

We will now define the surface of local equilibrium (curve 3 in Fig. 2) as the surface where the conditions $T_a = T_e$ and $\alpha = \alpha_E$ are satisfied, i.e., all forward processes are balanced by the respective reverse processes. This surface divides the shock layer into two regions. In the first region, which extends to the front of the shock wave (curve 1 in Fig. 2), ionization is the predominant process. In the second region, between the surface of local equilibrium and the body surface (axis $y = 0$, $y = (r-r_b)/r_b$), recombination prevails over ionization. It is evident here that, because of the rather low recombination rate, the values of α along the upper rays may exceed appreciably the corresponding equilibrium values α_E (see Fig. 1). Line 2 in Fig. 2 marks the position of the ionization front, plotted here as the surface of the maximum α gradient.

In Fig. 3a, b is shown the variation of the dimensionless normal velocity component u and of the transverse radiation flux q along theoretical rays through the shock layer. The variation of u ceases to be linear within the region of maximum α gradients. Along the upper ray ($\theta = 1$ rad), near the body surface, u changes sign and approaches zero from the positive side. As angle θ is increased, the maximum radiation flux decreases fast from its level in the quasiequilibrium region.

NOTATION

r	is the radius-vector;
r_b	is the radius-vector of the body surface;
r_w	is the radius-vector of the shock wave;
ξ	is the dimensionless radius vector;
θ	is the polar angle;
$\vec{V}$, u, v	are the velocity vector and its components along the radius-vector and normal to the radius-vector, respectively;
p	is the total pressure;
p_e	is the pressure of the electron gas;
ρ	is the density of the gas;
ρ_i	is the density of the ion gas;
α	is the coefficient of gas ionization;
α_E	is the coefficient of equilibrium gas ionization;
T_a	is the atom–ion gas temperature;
T_e	is the electron gas temperature;
m_a	is the mass of an atom;
m_e	is the mass of an electron;
e	is the charge of an electron;
A	is an atom in the fundamental state;
A^*	is an atom in an excited state;
A^+	is a single-charge ion;
ν_j	is the ionization frequency;
T_j	is the ionization temperature;
T_{ex}	is the excitation temperature;
R	is the specific gas constant;
k	is the Boltzmann constant;
h	is the Planck constant;
$\dot{n}_{ea}$	is the rate of ionization by an electron–atom collision;

$\dot{n}_{aa}$ is the rate of ionization by an atom–atom collision;
n_{rad} is the photoionization rate;
ω_{ie} is the rate of energy transfer during ion–electron collisions;
ω_{ae} is the rate of energy transfer during atom–electron collisions;
$\vec{E}$ is the polarization field intensity;
q is the radiation flux;
δ is the emissivity of the body surface;
M_∞ is the Mach number in the oncoming stream;
p_∞ is the pressure in the oncoming stream;
n_e is the electron concentration;
$n(1)$ is the atom concentration in the fundamental state;
$n(2)$ is the atom concentration in an excited state;
$n_a = n(1) + n(2)$;
$K_e(1, 2)$ is the excitation rate coefficient during an electron impact;
$K_e(2, 1)$ is the deexcitation rate coefficient during an electron impact;
$K_a(1, 2)$ is the excitation rate coefficient during an atom impact;
$K_a(2, 1)$ is the deexcitation rate coefficient during an atom impact;
g_1 is the statistical weight for the fundamental state of an atom;
g_2 is the statistical weight for an excited state of an atom;
g_i is the statistical weight for an excited state of an ion;
γ is the ratio of specific heats;
σ, C_i are constants.

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